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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In Quantitative Feedback Synthesis, bounds on plant uncertainty and on the system performance are specified. The minimum feedback is used which satisfies the latter over the range of uncertainty. Quantitative design has been extended to linear time invariant systems
(1) with nonminimum-phase (nmp) unstable plants with gain uncertainty. In the optimum design the gain factor uncertainty is maximized for which the specifications are satisfied;

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20. Abstract continued.

(2) parallel nmp plants whose output can be individually measured and processed, to achieve a parallel combination which is minimum-phase over the range of uncertainty, or if not possible, which is less strongly nmp.
(3) Cascade plants in which "plant modification" is possible by means of internal feedback. An added constraint is that the increase in plant interval variable c_i rms value ($\leq$) λ_i. Such internal feedback permits significant decrease in loop bandwidths and thereby the effect of sensor noise.
(4) A problem heretofore intractable to Quantitative Synthesis has been the Multiple Input-Output (multivariable) system. This problem has now been solved for plants with significant uncertainty and interaction. A remarkable feature is that the design procedure involves the design of a number of distinct, separate single-loop problems with no need for iteration. Constraints on the plant are less stringent than in other synthesis techniques which cannot handle significant parameter uncertainty.

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<u>ANNUAL REPORT</u>

FEEDBACK SYSTEM THEORY

AFOSR GRANT NO. 76-2946B

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for year ending October 31, 1978

Summary

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(4) A problem heretofore intractable to Quantitative Synthesis has been the Multiple Input-Output (multivariable) system. This problem has now been solved for plants with significant uncertainty and interaction. A remarkable feature is that the design procedure involves the design of a number of distinct, separate single-loop problems with no need for iteration. Constraints on the plant re less stringent than in other synthesis techniques which cannot handle significant parameter uncertainty.

FEEDBACK SYSTEM THEORY

1. Introduction

The primary purpose of ths research is the development of a Quantitative Theory of Feedback Systems. Feedback is mandatory when there is uncertainty in a system, but feedback theory has tended to be highly qualitative. Surely there should be a significant difference in the design of feedback system where plant parameter uncertainty is by a factor of say x%, and one where it is 100x% ? However, there are very few design techniques in the literature which reveal this. Our objective has been to develop such a quantitative theory step by step, starting with the simplest structures. In such a quantitative theory the problem statement includes bounds of the uncertain parameters and equally important, the tolerance on the system performance. The design should be related to these numbers and the final design which emerges should be tuned to the specifications.

Our progress this past year is best seen in the above context of development step by step, from simple to more complex structure, and from more to less restricted plant types.

2. Single Loop Linear Time Invariant System

In this class, the only remaining unsolved problem was that of uncertain plants with right half-plane poles and zeros. This problem was solved during the past year, for the case of gain factor uncertainty (with fixed poles and zeros). There always exists a range of gain for which the system poles can be constrained to be the left of $s = -\sigma$, for any $\sigma > 0$, or inside a circle of radius R , centered at $-(R+a)$, $a > 0$ for any a, R . The optimum design was defined as that for which the gain uncertainty, for which

the above constraints are satisfied, is maximized. A remarkably simple design procedure was found which gives the optimum design. The results have been submitted [1] and accepted for publication. There still remains the problem of nonminimum-phase (nmp) unstable plants with uncertain poles, zeros as well as uncertain gain.

3. Blending of Nonminimum-Phase Plants for Elimination or Reduction of Nonminimum-Phase Property

It is well known that right half-plane plant zeros severely restrict the bandwidth of the resulting loop, and thereby its sensitivity reduction properties. However, given two parallel uncertain nmp plant branches whose separate outputs may be sensed, then it may be possible to process these outputs such that the effective parallel combination is mp or at least weaken its nmp property - i.e., push the right half-plane zeros further away from the origin. An example is the longtitudinal axis in flight control with the elevator as the input and the pitch rate and acceleration as the two outputs. The latter branch is normally nmp and the former mp, but there are extreme situations where both branches are nmp. Using the results of Topic 1 above, this problem has been solved for the case of uncertainty only in the gain factors of the two parallel nmp plants. The case of uncertain poles and zeros was also studied and solved for one problem class. In the latter, the chief restriction is that the compensations introduced must have all their poles and zeros in the left half-plane. It is known, however, that superior results may be possible if this restriction is removed. So the most general case has not as yet been solved. The results have been submitted [2] and accepted for publication.

4. Multiple-Loop Systems with Plant Modification

In all our quantitative work to date and in other synthesis theories (state feedback etc), the feedback loops are returned to the plant input. The internal plant signal levels needed to achieve a specific output are then not a function of the compensations used. Thus, in Fig. 1a, $C_2 = C/P_1$, $C_3 = C/P_1P_2$ etc independent of the Q_i. This is a reasonable constraint corresponding to the practical case where the feedback specialist is not allowed to tamper with the insides of the plant. Feedback to internal plant variables makes the latter a function of the compensation, e.g., in Fig. 1b, $C_2 = \frac{C}{P_1}(1+P_1H_1)$, $C_3 = \frac{C}{P_1P_2}(1+P_1H_1)(1+P_2H_2)$ etc, a function of the H_1 compensations. But it may be permissible, providing the resulting increases in plant internal signal levels are constrained to be $\leq$ some specified $\lambda_i > 1$. An rms constraint was used and a synthesis theory developed for the cascade plant structure. A number of plant modification structures are possible. Two different structures are shown in Fig. 2 for the 3-sectron plant. A quantitative design procedure was developed for this problem. It was found that plant modification permits great economy in loop bandwidths. The results have been submitted for publication [3].

5. Design of Uncertain Multiple Input-Output Systems Reducible to that of Single Input-Output Systems

The latest [4] extension of quantitative design theory is to multiple (n) input-output systems, e.g., if $n = 3$, there are 9 system input-output response functions. In the linear time-invariant case, there is a 3×3 matrix of plant transfer functions with uncertain parameters. There are 9

closed-loop input-output functions, i.e., the 9 dynamic responses must be within specified bounds despite the uncertainty. Often one wants non-interaction, and this is easier to handle than the general case. The design technique can handle the general case. The remarkable features of the design technique are that

(a) it is exact - no approximations;

(b) design execution reduces to that of a number of single-loop non-interactive designs (even for the general case);

(c) there is no iteration necessary. The design is complete after the single-loop designs are done. Stability over the specified range of parameter uncertainty is automatically included with no additional effort;

(d) the limitations on the plant are less onerous than in other popular techniques which cannot handle significant parameter uncertainty. For example, Rosenbrook's [5] technique cannot cope with significant parameter uncertainties and requires either row or column dominance <u>for all</u> $\omega \in [0,\infty)$. Our technique copes with large parameter uncertainty and requires such dominance only as $\omega \to \infty$;

(e) Proof of the validity of the design theory involves abstract mathematics (Banach spaces and Schauder's fixed point theorem), but the design execution requires only frequency-domain concepts and tools, familiar to the practical control engineer. Only relatively elementary knowledge of matrix theory is needed. It is, in fact, remarkable that design theories with much less capability (i.e., which cannot cope with significant uncertainty and which cannot be extended to nonlinear

systems) require much greater knowledge of matrix theory - so much, in fact, as to place them beyond the grasp of most practical control engineers.

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3. I. Horowitz and Bor-Chyun Wang, Quantitative synthesis of uncertain cascade feedback systems with plant modification, submitted to Int. J. Control.

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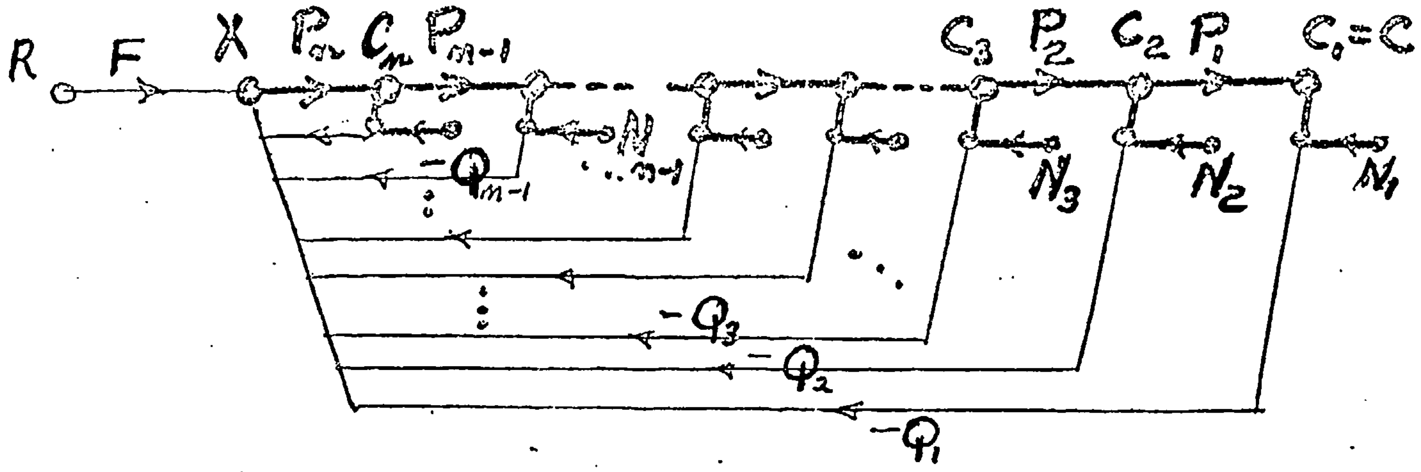


Fig. 1a No plant modification (NPM)

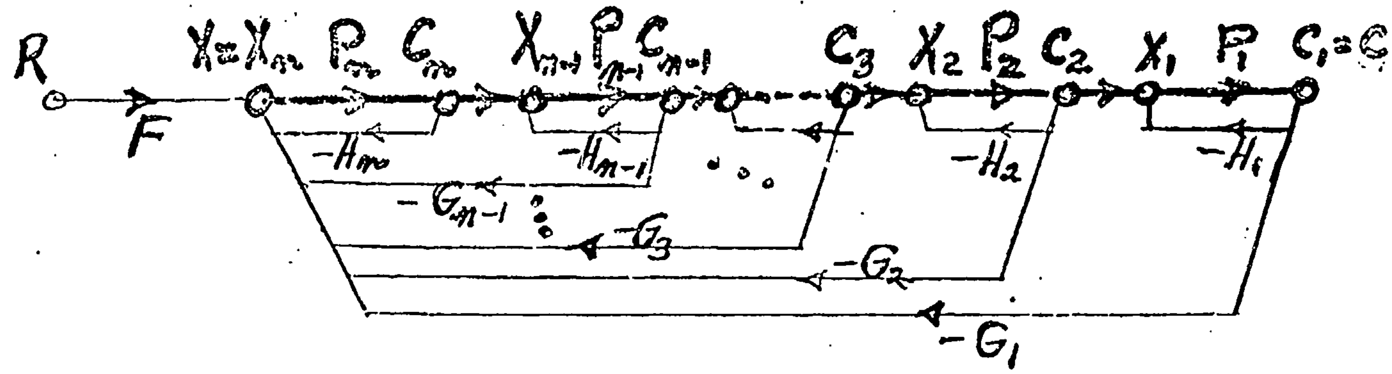


Fig. 1b Plant modification (PM)

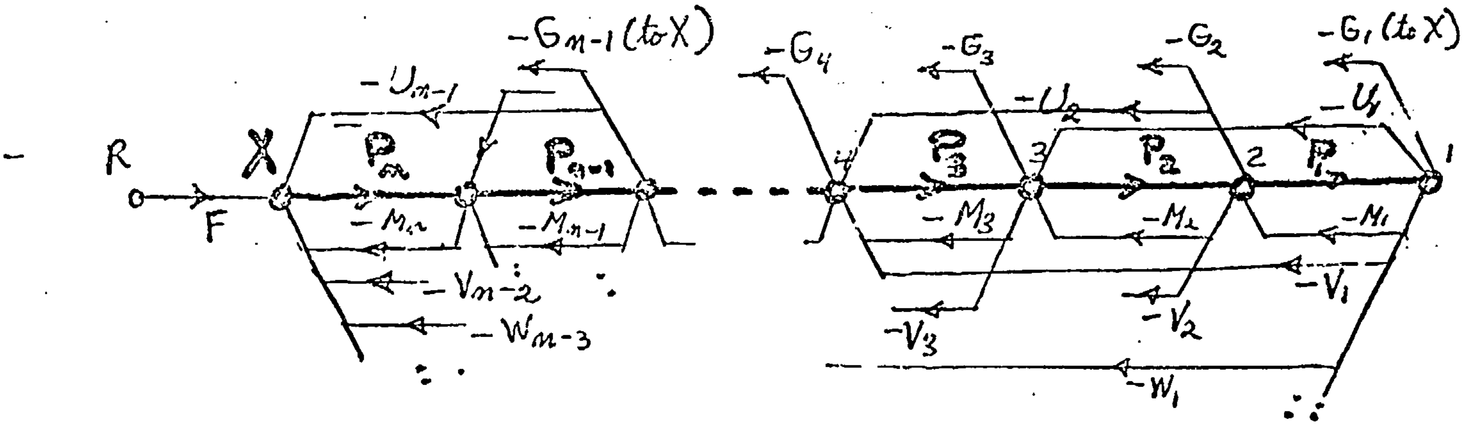
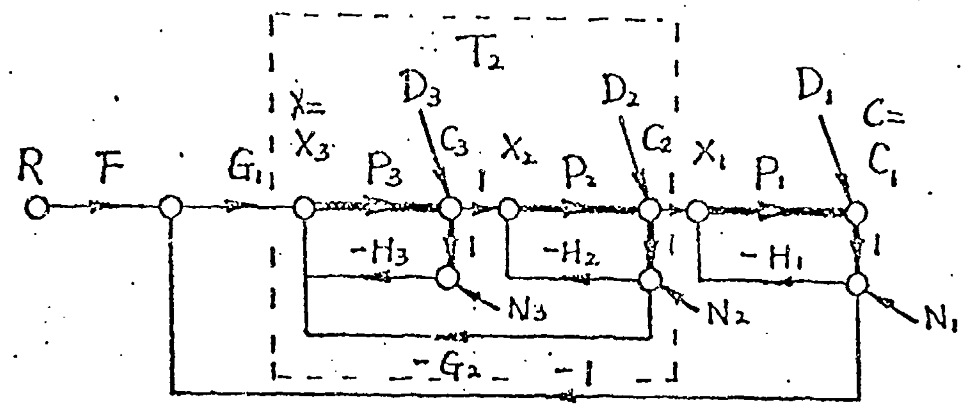
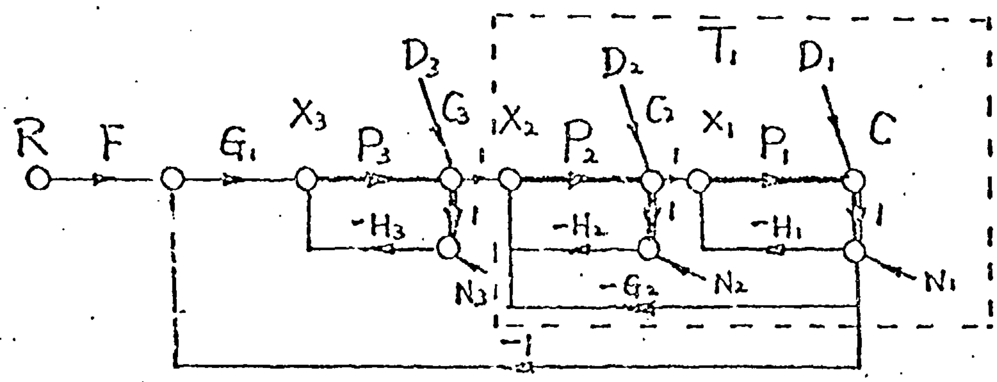


Fig. 1 Multiple-loop structures for cascade plant
Fig. 1c Plant modification



$L_{32} \triangleq P_{3e}P_{2e}G_2$, $T_2 \triangleq P_{3e}P_{2e}/(1 + L_{32})$, $L \triangleq T_2P_{1e}G_1$

(a) Structure I



$L_{21} \triangleq P_{2e}P_{1e}G_2$, $T_1 \triangleq P_{2e}P_{1e}/(1+L_{21})$, $L_o \triangleq P_{3e}T_1G_1$

(b) Structure II

$\ell_1 \triangleq P_1H_1$, $\ell_2 \triangleq P_2H_2$, $\ell_3 \triangleq P_3H_3$, $P_{1e} \triangleq P_1/(1+\ell_1)$,
$P_{2e} \triangleq P_2/(1+\ell_2)$, $P_{3e} \triangleq P_3/(1+\ell_3)$

Figure 2 5-loop, P.M. systems

Three section plant and its two (non-crossing) PM feedback structures